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EUROPEAN-STYLE OPTION MODELS

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Abstract: Currently, the stock market is developing all over the world, companies, various organizations and enterprises are putting their stock in options or shares. Of course, this can be risky, but in some cases it can be very useful. You can't predict whether it will be useful or not. In this paper, European-type options and their pricing models are studied, and an FMLS model for European-type option pricing is constructed using fractional partial differential equations.
Key words: Option, types of options, European type option price, European type option models, FMLS model

INTRODUCTION

Option means choice, right. An option is a contract. This on the basis of the contract, the buyer or the seller is given the right to buy or sell goods or securities at the agreed price within the set (agreed) period. The option is divided into buying option (option - call, call option) and selling option (option - put, put - option). In option-call, the owner has the right to buy, and in option-put, the owner has the right to sell.

There are European, American, Asian, Russian and other different types of options. In the European type, the term of the contract's expiration (extinguishment) is clearly set. In the American type, the time of the transaction (the time when the obligation will be fulfilled) is decided by the option holder. The holder of an American option has the right to buy at an arbitrary interval of the fixed term. Other types of options are also defined according to payment contracts. We will consider them in the next paragraph. The seller or the issuer gives the option buyer the right to buy or sell at an agreed price during or within the agreed period. But during this period, the price of the option may exceed or fall below the agreed price. If it falls, the recipient has the option of not taking it. In the market, the parties have equal rights, not equal opportunities. The seller or issuer has a certain risk. For equality, the buyer must pay for this opportunity, the seller must take the risk of falling prices, and receive a certain fee (fee) to cover certain losses. This fee (payment) will be the price of the option contract (transaction, agreement), that is, it will be the price of the option on the stock market. Therefore, we need to build a mathematical model of the stock market in order to estimate the option price. Based on all the obtained conclusions, it was studied how to obtain an exact closed-form analytical solution for European-type options through the first FMLS model by finding fractional derivatives.

FMLS model. Despite a number of difficulties, such as the non-locality of the spatial-fractional derivative, which prevents efficient discrete estimation of the option price, we have successfully developed an exact closed-form analytical solution for European-type vanilla options under the FMLS model. When using the newly obtained analytical solution, the asymptotic behavior of the solution can be well verified, which provides an additional basis for adopting the FMLS model for option pricing. In addition, we have confirmed the equivalence of call and put options under the FMLS model, both from a financial and mathematical point of view. Another important point is that the implementation of our solution is not as simple as the BS formula, because the integration core in the current solution involves Fox functions. However, we have proposed an efficient and accurate discrete estimation method to greatly facilitate the implementation of our formulation so that the FMLS model can be easily applied in commercial practice.

Under the risk-neutral measure, the FMLS model predicted that the dividend yield with the logarithmic value of $x_t = \ln S_t$ the principal fit the stochastic differential equation of the maximally skewed LS process:

$$dx_t = (r - D - \nu)dt + \sigma dL_t^{\alpha-1}$$

Here, r and D are the risk-free interest rate and dividend yield, respectively. t current time and $\nu = -\frac{1}{2}\sigma^{\alpha} sec\frac{\alpha\pi}{2}$ convexity coefficient. $L_t^{\alpha,-1}$ denotes the maximum skew of the LS process, which is a special case of the $L_t^{\alpha,\beta}$ Levy-α-stable process, where $\alpha \in (0;2]$ is the tail index describing the deviation of the Brownian motion of the LS process from the value, and $\beta \in (-1;1]$ is the skew parameter. To ensure that the main regression is supported along the entire real line, the tail index α should be limited to as shown in (2.3.1.3). We note that in the LS process with a maximum curve, random variable ($\beta = -1$) x_t is maximally left-skewed, that is, the right tail of the distribution quickly decays and exponential moments emerge. This parameter setting shows that FMLS only bounces downward, its upward movement has continuous paths, a feature that may not be consistent with empirical evidence. However, it should be noted that although this model is not perfect in modeling option prices, it can be seen as a springboard for future extensions to cover the finer features of the options market.

Below we look at the price of European-style vanilla options for this model. Let $V(x,\ t;\alpha)$ be the price of European-style options, x and $x = \ln S$ are the base price of the agreement, and α is the tail index. Cartea and del-Castillo Negrete showed that $V(x,\ t;\alpha)$ satisfies the following fractional differential equation under the FMLS model:

$$\begin{cases} \frac{\partial V}{\partial t} + (r + \frac{1}{2}\sigma^{\alpha}\sec\frac{\alpha\pi}{2})\frac{\partial V}{\partial x} - \frac{1}{2}\sigma^{\alpha}\sec\frac{\alpha\pi}{2}\ {}_{-\infty}D_x^{\alpha}V - rV = 0 \\ V(x,\ t;\alpha) = \prod(x) \end{cases} \quad (1)$$

Here, $\prod(x)$ is the payoff function, and $\max(e^x - K,0)$, $\max(0,e^x - K)$, and K are the purchase price for European-style call and put options, respectively. ${}_{-\infty}D_x^{\alpha}$ is a one-dimensional Weyl fractional operator defined as:

$${}_{-\infty}D_x^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)}\frac{\partial^n}{\partial x^n}\int_{-\infty}^{x}\frac{f(y)}{(x-y)^{\alpha+1-n}}dy,\quad n-1 \le \Re(\alpha) < n$$

From this formula, it can be seen that when $\alpha \to 2$, the above one-dimensional Weyl fractional operator becomes a second-order differentiation, as a result, formula (1) reduces to classical BS systems for European options.

It should be noted that the fraction in the formula (1) is fundamentally different from partial differential equations, where the derivative of the fraction appears in the direction of time and can be eliminated by the Laplace transform, while the Laplace transform does not help us. Despite these difficulties, we were able to obtain an exact closed-form analytical solution from formula (1), as shown in our subsequent work.

In finance, as the convexity adjustment is introduced into the Levy process, there is a measure of risk aversion under the FMLS model as mentioned above. The existence of a risk-neutral measure, on the other hand, means that the assumption of "no arbitrage opportunity" still holds in this model. Put-call parity can be achieved using the portfolio analysis adopted in the BS model.

On the other hand, through rigorous mathematical analysis, it can also be shown that put-call parity exists under the FMLS model, as expected from the aforementioned financial argument.

Summary. The constructed regression model passed T-test and F-test very well with 95% confidence. The coefficient of determination of the model is also 80.2%, that is, 80.2% of the change in the volume of TTXI can be explained by the change in real interest rates, the number of labor force in the country, and the change in the country's gold and currency reserves. It is possible to describe the built model for easy use in practical issues.

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