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Analysis of Methods for Solving Optimal Control Problems Train Movements

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Abstract: *Optimal train management has been an active research topic for many years. All over the world, important measures are being taken to use electricity efficiently due to the growing demand for energy resources. Such measures are also being taken in the railway industry. The overall objective is to operate the train in such a way as to minimize the overall energy consumption, taking into account the time and physical constraints imposed by the train and the operating conditions. The achievement of this goal is ensured by advanced technologies currently used on electric rolling stock and new computer capabilities. The lion's share of electricity consumption in railway transport is accounted for by the cost of train traction. Therefore, measures aimed at optimizing electricity consumption by electric locomotives and electric trains are relevant. Optimization is a powerful tool and a promising solution to any problems associated with the operation of railway transport. The ever-increasing complexity of engineering systems, the growing demand for accuracy and the search for optimal and reliable designs create additional difficulties that can only be solved by developing optimization models. In theory, optimization is the process of maximizing or minimizing an objective function by sequentially selecting and calculating possible results within a certain set of parameters. In the context of growing competition in the transportation market, the interest in energy efficiency among railway companies in recent years has become the subject of increased interest, both for the modernization of existing vehicles and for the acquisition of new ones.*

Keywords: *Energy efficiency, optimal train movement control, the optimal trajectory planning, energy-optimal movement trajectories, dynamic programming.*

Train traffic is subject to several uncertain factors such as unexpected stops and mechanical problems. These factors affect schedule compliance and energy efficiency. One way to eliminate these problems is to determine the energy-optimal train trajectory between two stations within a predetermined time, taking into account uncertain factors.

The problem of finding a certain curve of motion, i.e. sequences of velocity values along the time and path axis arises as an optimal control problem, taking into account certain operational, geographical and physical constraints. The main goals of the optimal motion trajectory considered so far are related to the solution of the following problems:

1) timely arrival at the destination, i.e. deviation from the specified time should be kept to a minimum;
2) the shortest travel time, i.e. travel time should be kept to a minimum and speed should be as high as possible;

3) minimum power consumption, i.e. overall electricity consumption should be kept to a minimum.

Because all three of these goals conflict with each other, many studies focus on their combination or compromise, or include one of them as a constraint and the other as a goal.

Research into optimal train curve planning began in the 1960s. A simplified problem of optimal train control was considered in [26,34]. The authors solved this problem using the Pontryagin maximum principle. Later, many researchers solved the problem of optimal control [2,3,6,10,11,13,20] by applying various methods, since it has a significant impact on energy savings, punctuality and driving comfort. The classification of energy-optimal calculation methods is shown in Fig.1.

Known approaches to solving problems of determining the optimal energy-consuming trajectory of trains can be divided into two groups: analytical and numerical.

The analytical algorithm requires good objective function properties, so researchers have to simplify some conditions when modeling. The numerical algorithm does not impose any requirements on the objective function. However, the analytical algorithm allows you to accurately obtain the optimal solution, even if the process is complex. For a numerical algorithm, there is a trade-off between precision and computational efficiency. As a rule, the calculation speed is not high, and sometimes it can only find a local optimal solution. But accuracy can be guaranteed by using some numerical solvers with sufficient computation time. The premise is that the problem of energy-optimal train driving is formulated as a model of mixed integer linear programming with some approximations.

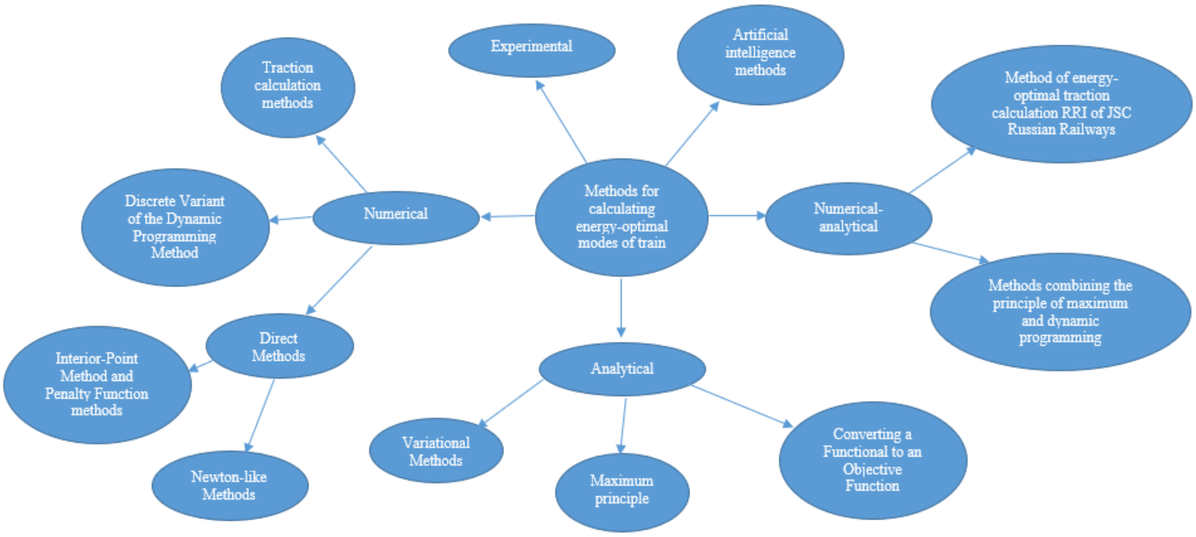

Fig.1. Classification of energy-optimal calculation methods

The energy-efficient method of driving a train was primarily based on the theory of optimal control. Thus, at first, for simplicity, the problem was formulated in the form of continuous optimal control models based on the classical calculus of variations. Y.P. Petrov [14, 15]. The optimality conditions were formulated by him in the form of the Euler equations. Provided that the traction force can vary continuously, and the efficiency of the locomotive traction drive is constant, it was found that the optimal motion curve usually consists of sections in which the speed should be constant, and sections with uneven profiles corresponding speed restrictions [17]. Later, scientists actively developed algorithms for optimizing control modes, where the theoretical solution was carried out using the maximum principle of L. S. Pontryagin. The Pontryagin maximum principle, in contrast to the classical variational calculation, makes it possible to solve control problems in which restrictions are imposed on the control parameters, although a number of properties of the solution are usually specified in advance. Due to this, the maximum principle is the main mathematical technique used in the calculation of optimal control in many important engineering problems [12,16].

I. A. Asnis et al. [1] assumed that acceleration is a continuous control variable with uniform boundaries, and used the Pontryagin maximum principle to find the necessary conditions for the optimal motion schedule. In [33], the author proposed a nonlinear second-order model for minimizing electricity consumption, where the Lagrange problem was solved. To search for more rigorous mathematical proofs, Howlett [30,31] showed that the problem can be formulated in the corresponding function space. He came to the conclusion that an optimal motion schedule exists and that the motion schedule must satisfy the criterion of Pontryagin's principle. Y.M. Golovicher [24] proposed an analytical method for optimizing train operation with minimal energy consumption. To reduce computational time, the Hamiltonian formulation with the maximum principle was applied to determine the optimal operating mode. It has been found that optimal control can save 3% of energy consumption.

Depending on whether the traction and braking force is continuous or positional (discrete), there are two options to solve. One of the options is designed for operations with trains with continuous control, while the other is for operations with trains with positional control [32]. Based on the analytical approaches mentioned above, there are four optimal sequences of control scenarios on the optimal driving curve: acceleration with maximum acceleration, movement at a steady speed, coast down and deceleration with maximum deceleration. However, a more detailed model of train operation is considered in [36], which includes the efficiency of the propulsion system and the scheme of regenerative braking of electric rolling stock. It is worth noting that analytical methods often encounter difficulties in finding an analytical solution, given the more realistic conditions that introduce complex nonlinear terms into the model equations and constraints.

Numerical optimization methods. Y.M. Golovicher [37] stated that classical numerical optimization methods are not suitable for solving the optimal motion curve planning problem on an on-board computer for real-time calculations, since these methods, such as discrete dynamic programming, require significant computation time. Therefore, research in this area has long been hampered by computational difficulties. However, due to the high computing power currently available, more and more researchers are applying approaches to obtain energy-optimal motion curves using numerical optimization methods.

Given the complex relationship between train movement and energy consumption, analytical results are only available in simplified cases. Because of this difficulty, much of the effort has been devoted to the development of approximate numerical schemes and algorithms. Here we briefly mention some of the most important approaches of numerical methods [19] applied for optimal control, undertaken in the context of optimizing railway systems.

Direct methods. Direct methods do not require prior knowledge of the structure of the solution. The first step is to discretize the problem to get a finite dimensional problem and then non-linear programming techniques can be used. The idea of these methods is to solve simpler subproblems that converge to the original solution in a finite number of iterations or in the limit. Two different types of algorithms are considered:

1) Interior point and penalty function method: the problem is reformulated to turn it into an optimization problem without restrictions. Thereafter, unrestricted optimization methods, such as gradient-based methods [29], can be used to find a solution.

2) Newton's tangent method. The problem is solved by finding a point that satisfies the Karush-Kuhn-Tucker conditions (necessary conditions for optimality). In [21], quadratic programming was used to solve a simplified train model.

Approach to the solution with dynamic programming. Dynamic programming methods allow to solve the control problem without any initialization of the problem and under any given circumstances an optimal solution can be found, this is one of the main advantages of the dynamic programming approach, it goes through the entire state space to provide solutions from any possible state space point to destination. The idea is to divide a complex problem into simpler subproblems, and each time a subproblem is solved, the solution is stored in memory to help solve the larger subproblems. The main disadvantage of using this method is that it involves very expensive computational costs.

Currently, computing power has increased significantly compared to the period when most of the work was written. Therefore, in [36], a more detailed nonlinear train model is proposed, in which the power losses of an electric locomotive with a traction converter are simulated. The problem of planning the optimal motion curve based on this nonlinear model is solved by methods of nonlinear programming and dynamic programming. It is concluded that discrete dynamic programming turned out to be more effective for solving a nonlinear optimal problem compared to sequential quadratic programming, since the total computation time of discrete dynamic programming is deterministic and the calculation result is obtained in the form of a feedback control law.

Among the numerical methods for solving the problem of optimal control with dynamic programming, the method of dynamic programming by R. Bellman [4] is widely used. The Bellman method is based on the principle of optimality “the optimal control strategy has the following property: whatever the initial state and the decision at the initial moment, the following decisions should constitute the optimal control strategy with respect to the state obtained at the initial stage of the process” [5]. This method allows us to formulate simpler algorithms for optimizing dynamic objects of small size [7–9,18]. In [25], dynamic Bellman programming is used to optimize the optimal basic trajectory.

Dynamic programming, the gradient method, and sequential quadratic programming are introduced to solve the optimal motion curve planning problem in [28]. In simple and complex operating conditions, simulations have shown that the gradient method has good convergence.

However, the optimal solution is not always guaranteed in these numerical optimization approaches. Since the resulting "optimal" solution may be a local minimum. In addition, the rate of convergence is generally uncertain. Moreover, the computations of these numerical optimization approaches are often too slow for real-time applications.

Fuzzy and evolutionary algorithms. To achieve a certain effect on the motion curve in energy efficient solutions, some fuzzy and intelligent methods have been introduced and improved, including the genetic algorithm [29], swarm intelligence [35] and neural network [42]. However, in addition to providing energy efficiency, it is expected to achieve a complex effect in relation to several aspects based on a significant level of energy capabilities, which means that the adequacy, accuracy of the thrust model and the efficiency of the calculation of the solution should also be taken into account. S. Yasunobu [43] proposed a fuzzy automatic train operation controller and implemented it in the Japanese city of Sendai in 1987. This controller can manage each train's departure, speed regulation and waiting time. The membership function plays an important role in ensuring the control accuracy and reliability of the fuzzy automatic train operation controller. Therefore, in [22], the authors proposed a modified differential evolution algorithm for optimal tuning of fuzzy functions that provide a compromise between punctuality, ride comfort, and power consumption. The implementation of a genetic algorithm for optimizing train control was demonstrated in [23]. In the paper, the results are tabulated for a line of control commands that are referenced by the train's automatic operation system to decide when to coast and resume maximum acceleration. Khan [39] also uses a genetic algorithm to construct the optimal baseline motion curve. Yu. Bocharnikov et al. [47] concluded that energy savings are affected by accelerations and decelerations by running a series of simulations in parallel using a genetic algorithm. In [38], the authors combined artificial neural networks and a genetic algorithm to obtain an optimal coastdown schedule. The objective function is considered as the total power consumption and the efficiency of the computing system.

Optimization of the distribution of section travel time for travel times along the stage and energy-optimal guidance, the two main used energy-optimal train movements to minimize power consumption, have been studied for a long time. Optimization of the distribution of section travel time for travel times along the stage calculates how to compensate for the delay of the train by reducing the travel time along the stage on the remaining stages. When the train is ahead of the planned movement schedule, it is calculated how to distribute the excess time between the stages in front in order to arrive at a given station at a fixed time, and also solves the problem of maximizing the use of regenerative energy based on the acceleration and deceleration time according to the movement schedule. Energy Optimal Guidance - optimizes traffic schedules on sections to minimize traction power consumption in accordance with speed and travel time limits. Optimization of divisional time distribution and energy-optimal regimens are interrelated. The former provides travel times on each stage for the second, and the latter optimizes the acceleration, coastdown and deceleration times on each stage for the first. Energy-optimal train guidance is aimed at optimizing the travel curve between two stations for one train and in many problems ignores the regenerative energy transferred back to the contact network. Therefore, the resulting energy-optimal motion curve is optimal for only one train. Optimizing the distribution of section travel times to travel times across the span synchronizes the actions of several trains to maximize the use of regenerative energy, but usually assumes the time schedule as a constant parameter. Traction effort from the obtained optimal distribution of the segment travel time is not reduced. Therefore, in recent years, many researchers have been studying the method of complex optimization.

In [27], the regulation and coordination of the operation of several trains in real time with mixed control of the sectional running time is described. The goals are minimum energy consumption and comfort. To find a solution, the authors used a dynamic programming approach. A slightly different scheme for solving the same problem, also based on the dynamic programming method, is described in [19]. Yu.V. Bocharnikov [49] presented a model for optimizing the train schedule, taking into account both the optimization of traction and the use of regenerative energy. He also ran simulations to evaluate the benefits and results of an optimal driving schedule while minimizing energy consumption. In [48], the problem of energy-optimal operation of trains was formulated as a two-level optimization model and a genetic algorithm was developed to find the optimal solution. At the first level, a suitable trajectory for the passage of the section for trains was determined, and at the second level, the travel time for each section was determined in order to minimize the energy consumption for traction. In [40], the authors proposed an integrated train control model to reduce energy consumption and developed a numerical algorithm for obtaining optimal driving modes with a given travel time, which takes into account the variable resulting forces acting on the train, speed limits and slopes. H. Yang [45] developed a comprehensive optimization method to reduce overall power consumption and overall travel time. In the work, the author finds the optimal time of arrival of trains at the stations and the maximum sectional speed of the train in sections along a certain movement curve. There are works devoted to integer programming models for determining the schedule and curve of movement with minimum power consumption, where regenerative energy is taken into account [44]. There, a comparison was made between the method of optimizing the distribution of segmental travel time [46], the energy-optimal method of conducting [41], and the complex method of optimizing energy consumption. The results showed that the integrated optimization method can reduce the overall energy consumption compared to other methods.

Various methods in the literature are grouped into two main categories: analytical solution and numerical optimization. As stated above, analytical methods often encounter difficulties in finding analytical solutions when more realistic conditions are considered that introduce complex non-linear terms into the equations of motion and constraints. Most of the methods have shown their effectiveness in numerical examples, but only a few of them have been tested in real systems. In the laboratory simulation, the trains always adhere strictly to the given timetable and arrive at each station on time in strict accordance with the schedule. However, in practice, trains may have some slight deviations. These small deviations do not affect normal operation, but they do have some impact on energy estimates. Therefore, more empirical studies should be carried out to test its effectiveness in the practical operation of railway systems.

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