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The Lyapunov Construction Function for a System of Simple Differential Equations

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Abstract: *In this article, we study the behavior of the construction of the Lyapunov function for one system of differential equations. There is only one whole trajectory on the Ox axis – the singular point 0(0;0).*

Keywords: *closed, zero solution, coordinates, identity, stability, asymptotes, set, trajectory.*

The viewing system is advanced

$$\begin{cases} \frac{dx}{dt} = \varphi_1(x) + \psi_1(y) \\ \frac{dy}{dt} = \varphi_2(x) + \psi_2(y) \end{cases} \quad (1)$$

Assume that the right portions of the system (1) $\varphi_i(x), \psi_i(y)$ $(i = 1,2)$ are continuous in some open area D, which can coincide with all planes, and that the functions $\varphi_i(x)$, $\psi_i(y)$ satisfy any closed domain G lying in D to the Livshitz conditions. Assuming that the requirements $\varphi_i(0)$, $\psi_i(0) = 0$ are met, the point 0(0;0) will be a unique point of the system (1), or, which is also the zero solution of the system. Let's attempt to locate the Lyapunov function for it in the shape.

$$v(x, y) = F_1(x) + F_2(y)$$

The area D, which encompasses the origin of coordinates, contains the continuous partial derivatives of the functions $F_1(x), F_2(x)$, and the function $v(x, y)$is one of those functions. Using system (1), the derivative function $v(x, y)$ with respect to time $\dot{v_t}$ will assume the following shape.

$$\dot{v_t} = F_1'(x)[\varphi_1(x) + \psi_1(y)] + F_2'(y)[\varphi_2(x) + \psi_2(y)]$$

We demand once more that the equality holds and that the function $\dot{v_t}$ has the same functional structure as the function $v(x, y)$.

$$F_1'(x)\psi_1(y) + F_2'(y)\varphi_2(x) = 0$$

By dividing the variables we get

$$\frac{\varphi_2(x)}{F_1'(x)} = -\frac{\psi_1(y)}{F_2'(y)}$$

and each of these parts must therefore have a fixed number, such as $\frac{1}{c}$. So, we obtain:

$$F_1(x) = C\int_0^x \varphi_2(\xi)d\xi \ , \qquad F_2(y) = -C\int_0^y \psi_1(\eta)d\eta$$

$$v(x,y) = C\left[\int_0^x \varphi_2(\xi)d\xi - \int_0^y \psi_1(\eta)d\eta\right]$$

The formula for the derivative $\dot{v}_t$ is obtained.

$$\dot{v}_t = C[\varphi_2(x)\varphi_2(x) - \psi_1(y)\varphi_2(y)]$$

One can find the necessary conditions for the stability of the zero solution of the system using the signatures of the functions $v(x,y)$ and $\dot{v}_t$. (1).

We use an illustration of this idea.

$$\begin{cases} \frac{dx}{dt} = ax^{2m+1} + by^{2m-1} \\ \frac{dy}{dt} = -Cx^{2m-1} + ey^{2m+1} \end{cases} \qquad (2)$$

here *a*, *b*, *c*, *e* - constant coefficients, and $b \cdot c \neq 0.$

Теорема 1. Если 11 $b \cdot c < 0$, то нулевое решение неустойчиво.

Этот факт доказывается применением метода Фромера. Введем подстановку 22 $y = ux^{A}$ и найдем 33 $\boldsymbol{\lambda}$=1, то есть вводя подстановку 44 $y = ux$ получим.

Theorem 1. If $b \cdot c < 0$, then the zero solution is unstable.

This fact is proved by using Fromer's method. We introduce the substitution $y = ux^{A}$ and find $\boldsymbol{\lambda}$=1, that is, by introducing the substitution $y = ux$ we get.

$$x\frac{dy}{dx} = \frac{-(C+bu^{2m})+x^2(eu^{2m+1}-a)}{bu^{2m-1}+ax^2} \qquad (3)$$

We determine from the system

$$x = 0, \ c + bu^{2m} = 0 \qquad (4)$$

singular points correspond to exceptional directions.

$$y = x\sqrt[2m]{-\frac{c}{b}x}$$

Since the singular points derived from (4) will be saddles that the identity D, systems (2) that exist for $b \cdot c < 0$ and these orientations will be of the second type.

When $b \cdot c > 0$, or the single point x = 0 , y = 0 system (1), is of the center type, the stability issue emerges.

The $F_1(x), F_2(y)$ functions for system (2) will have the following form:

$$F_1(x) = Cx^{2m} , \qquad F_2(y) = by^{2m}, \qquad v(x,y) = Cx^{2m} + by^{2m}$$

For the derivative $\dot{v}_t$ we obtain the expression

$$\dot{v}_t = 2aCx^{4m} + 2bey^{4m} \quad (c = const = \frac{1}{2})$$

Theorem 2. If $a \leq 0,\ e < 0,\ bc > 0,$ then the zero solution is asymptotically stable.

The proof has two parts.

1. If we assume that $a < 0,\ e < 0,\ bc > 0,$ then we are in the conditions of application of Theorem 4.2. [1]. The zero solution will be asymptotically stable.
2. If $a = 0,\ b > 0,\ e < 0,\ c > 0,$ then the zero solution will be stable, as follows from Lyapunov's theorem. However, in this case, from Theorem 5.2. [1] follows in the asymptotic stability. Indeed, the set M in Theorem 5.2. [1] serves the set y = 0, that is, the x-axis.

However, it is clear that the Ox axis does not contain the complete trajectory of the system (1). In fact, if such a track was on the Ox axis, we would have $y = 0$ и $\dot{y} = 0$ and we would be able to determine that x = 0 from the second equation of system (1). Consequently, there is only one spot on the complete trajectory, which is 0(0;0).

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